# Modeling Comparisons for some Classification Methods, Bayesian, Neural and Traditional Cluster Techniques

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**Modelado de comparaciones para técnicas de clasificación bayesianas, neurales y técnicas de conglomerados tradicionales**

**Resumen.** Se comparan algunas técnicas de clasificación muy útiles para análisis de conglomerados en *marketing*. Los primeros están basados en la Modelización Mixta de Clases Latentes con datos de entrenamiento y posteriormente sin ellos. El segundo conjunto de técnicas se fundamenta en los métodos de clasificación basados en las redes neuronales. Finalmente, se presentarán métodos más clásicos basados en las técnicas de K-medias seguido de los conglomerados jerárquicos.
**Palabras clave:** técnicas de segmentación, modelización con clases latentes, datos de entrenamiento, clasificación *K*-medias, método de clasificación jerárquica.

**Abstract.** This article compares some classification methods that would be very useful for clustering purposes mainly in marketing. First of them are based on Latent Class Mixture Modeling with training data and without training data. The second set of techniques is based on Neural Networks Classification Method and finally we will present methods based on more classical techniques like K-Means and Hierarchical Cluster Analysis techniques.
**Key words:** segmentation techniques, latent class modeling, training data, neural networks, *K*-means classification method, hierarchical classification method.

## Introduction

It is not unusual to see market investigations based on subjective criteria like perceptions, interests, characteristics of personality, behavior, etc. There is still some difficulty analyzing unobservable variables, which could be analyzed in an indirect way, this kind of unobservable variables are fundamental for the clarification of many marketing concepts or constructs that we can also call latent variables and are becoming very important in identifying market segments.

The procedures of latent class models, also known as *Mixture* Modeling, could perfectly comply with market segmentation and allow to identify a mutually exclusive set of groups also called latent class that explain the similarity of cases measured by a set of variables. These techniques classify groups, which sizes are *a priori* unknown or just some cases are known. There is some relation between latent class modeling and some non-hierarchical procedures like k-means clustering because both optimize some kind of criteria (log-likelihood and intra-group variance).

The goal of this article is to present and compare new techniques of classification suitable for social sciences and particularly such areas that use cluster analysis and segmentation like marketing, sociology, strategy, but also other areas as experimental sciences we can see in the next example. We will present some segmentation techniques close to latent modeling, *Mixture* Modeling with and without data training.

Some other techniques like neural networks are very precise finding the posteriori cluster, these methods are not new and they offer very good results. We will use the trained multiplayer neural network with back propagation perceptron algorithm and conjugate gradient descent that will use an inner layer of variables, some intermediate layers and the exit layer with the results.

Finally we will use traditional clustering techniques like *K*-means and hierarchical cluster analysis to assign cases to groups. For all analysis we have been using the example of Anderson (1935).

## 1. Classification with Latent Class *Mixture* Modeling

*Mixture* Modeling has awakened a very sustainable interest these last years because it is based on a statistical model for the whole population, it permits to work on a sample using all traditional optimization tools; this means that the observed data are generated as a result of a concrete finite *Mixture* of probability distributions.

*Mixture* models refer to a class of procedures that provide a simple and effective approach to modeling population heterogeneity, the term *Mixture* is used because the population is assumed to consist of homogeneous subgroups. This modeling allows making a probabilistic fuzzy classification based on ex-post models of classification, the technique gives a probability of classification for each data, the total probabilities being equal to 100% (Lilien and Rangaswamy (1998).

Finite Mixture Modeling is starting from the assumption that a sample of observations are coming from different groups mixed with unknown proportions. The goal is to identify the different samples estimating the post density function parameters (Wedel and Kamakura, 1998).

The first step is to assume that observations (subjects) come from a population, which is a *Mixture* of $S$ segments or groups that are categories of the latent variable, $C_1$ to… $Cs$ in proportions $p_1$ to,…, $ps$. It is not known *a priori* from which segment a particular observation comes.

$$\sum_{s=1}^{S} \pi_s = 1, \ \ \pi_s > 0, \ S = 1,..., S$$

The distribution function of the observed variables vector $y_n$ could be read in its general form $f(y_n | \varphi_s)$. Here, $\varphi_s$, represents a vector with all unknown parameters associated with the density function selected for each class S.

The conditional density function $f(y_n | \varphi_s)$ could be presented as an exponential type function

$$f_i(y_{nj} \mid \theta_{nj}, \lambda_j) = \exp\left(\frac{y_{nj}\,\theta_{nj} - a_j\,(\theta_{nj})}{j \quad \lambda} + \mathrm{b}_j(y_{nj}, \lambda_j)\right)$$

where $\varphi_s = (\theta_{nj}, \lambda_j)$. $\theta_{nj}$ is the canonical parameter and $\lambda_j$ is the dispersion parameter for the s group.

The second step presents the latent class general model and specifies the unconditional distribution of the $y_n$ where $\Phi = (\pi, \varphi)$. The observed distribution $f(y_n|\Phi)$ is a *Mixture* of different densities $f(y_n | \varphi_s)$ with different proportions $\pi_s$.

$$f_i(y_n \mid \Phi) = \left(\sum_{s=1}^{S} \pi_s f_s\,(y_n \mid \varphi_s)\right)$$

Normally if no restrictions are imposed, the problem should be resumed to estimating a set of independent parameters for each latent class, the most common optimization method used for the parameters estimation is the Maximum Likelihood, and the log-likelihood required function is:

$$L(\Phi \mid y_1,\ldots\ y_n) = \sum_{n=1}^{N} L_n\left(\sum_{s=1}^{S} \pi_s \ \ f_s(y_n \mid \varphi_s)\right)$$

The last step should be to classify individuals or observations in different groups, for that we compute the a posteriori probability to classify each subject or observation according the Bayes rule (see Lévy Mangin 2006, Micah Altman, Jeff Gill, Michael, P. McDonald, 2004, Picon Prado, Varela Mallou, Lévy Mangin, 2004, Kamakura and Wedel, 1998).

### *1.1 Classification with Mixture Modeling and Training Data*

The classification with *Mixture* Modeling and training data´s model is appropriate when a model is not suitable for an entire population and you have to divide it into subgroups (Loken, 2004, Vermunt and Magidson, 2005, Lee, 2007). The data chosen for the whole article come from the Anderson´s (1935) example. The data set contains four measurements of flowers from 150 different plants, the first 50 flowers are irises of the species *setosa*, the next 50 are irises of the *versicolor* species and the last 50 are irises of the *virginica* species.

The chosen model is a confirmatory factor analysis model with correlation of all observed variables (figure 1), the Petal length, the Petal width, the Sepal length and the Sepal width, in which we will perform a Bayesian analysis to compute the posterior probability of belonging to each group for all flowers.

Once the *Mixture* Modeling classification done, we have performed a comparison with the *a priori* classification and made a cross tabulation that makes a comparison between the *Mixture* Modeling classification and the *a priori* or real classification.

It could be observed in the cross tabulation table (see table 1A in appendix) that 50 cases have been correctly classified (100%) for *setosa* (1), 46 (92%) for *versicolor* (2) (4 have been wrongly classified) and 49 (98%) for *virginica* (3) (1 has been wrongly classified).

After processing a discriminant analysis (see table 2A in appendix), it is interesting to observe that all *setosa* species have been correctly classified; the discriminant analysis counts for 47 *versicolor* species correctly classified when there are 50 in the original classification and 51 *virginica* were classified when there are only 50. We can observe that the discriminant analysis has classified in base of the *Mixture* Modelling classification and not on the base of real classification. For that reason we have three cases of misclassification, two predicted in the *virginica* species when they are versicolor, and one case classified and predicted *virginica* when it should be classified and predicted versicolor.

In conclusion *Mixture* Modeling with training data supports a 100% of original *setosa* classification, 92% of original *virginica* classification and 98% of original *versicolor* classification.

### *1.2 Classification with Latent Structure*

The Latent Structure Analysis is a variant analysis of *Mixture* Modeling where the indicators should be independent within each group and uncorrelated when they are multivariate normally distributed (Lazarsfeld and Henry, 1968) as shown in figure 2.

Latent Structure models will have a special particularity, observed variables will not be correlated each other and the data will not be trained; this suppose that there is no *a priori* classification for some observations and it could be anticipated that the algorithm should have more difficulty classifying data or finding the (posterior) classification probability for each species.

Table 3A (see appendix) shows that latent Structure without correlation and training classifies correctly the *setosa* species (100% correct, 50), quite well the *versicolor* species (94% correct, 47) and *virginica* species (92% correct, 46). This classification could be slightly less precise than the *Mixture* Modeling with correlation and training classification, lack of correlation between observed variables (indicators) and data training have made the difference.

The discriminant analysis (see table 4 in appendix) classifies correctly the *setosa* species (100%), the *versicolor* species (100% but it identified 51 cases instead of 50) and the whole majority of *virginica* species (48 over 50, 98%).

## 2. Neural Networks Classification with the Multilayer Perceptron Method

The Multilayer Perceptron method with the back propagation activation function is a current method of classification used by the Neural Networks and it is probably one of the most popular and most extensively used (Haykin, 1994, Bishop, 1995, Picon, Varela, Lévy Mangin 2004) in Social Sciences, particularly in marketing. We will use this method to classify the output units using the Statistica Neural Networks program, which selects automatically the best network among many.

The inner units or inputs (4) go into an activation unit through a transfer function to produce an output (3). The number of inputs (here the Petal length, the Petal width, the Sepal length and the Sepal width) and output units (here *setosa*, *virginica* and versicolor) is defined by the problem. The number of cases assigned to the training process is 80 over 150 cases. The number of hidden layers (one with 8 intermediate units) has been selected in order to minimize the prediction error.

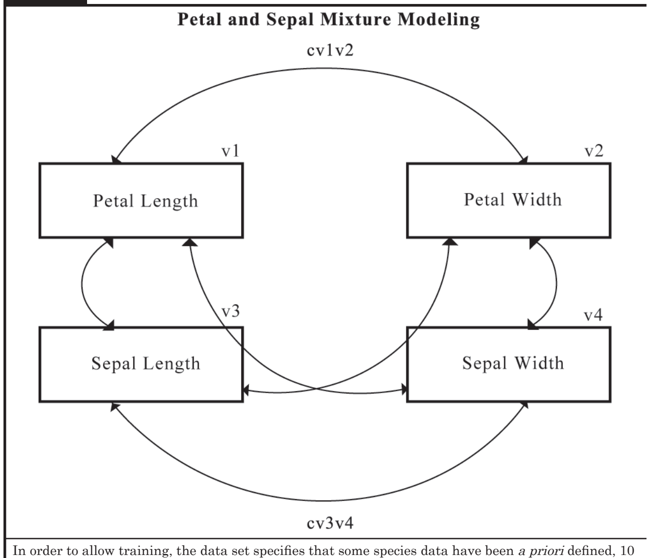
The output of each node of the intermediate layer should be estimated by the next function where

$$Z_j = \varphi \sum_{i=0}^{n-1} W_{ij} X_i$$

$\varphi$ represents the node transfer function, $W_{ij}$ the weights between nodes $_i$ and $_j$, Xi the inner units or input and $Z_j$ is the exit of the $_j$ node. The difference between the node input and the output exit is the error. In our case, a network of 1 hidden layer has generated the best solution with 8 units, as summarized in figure 3.

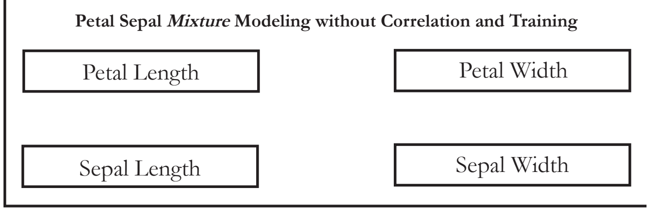
As shown in figure 4, the error training for this network is 0.002442, for the verification set 0.1003 and the global performance is 0.9857143; which is very good.

**Figure 1.** Mixture modeling with training data and all correlations.




In order to allow training, the data set specifies that some species data have been *a priori* defined, 10 for *setosa* (1), 10 for *versicolor* (2) and 10 for *virginica* (3).

**Figure 2.** Latent structure without correlation and no training.



The Multilayer Perceptron Neural network was the most accurate method; it made just 1 error classifying 1 flower *versicolor* in the *virginica* species (see table 5A in appendix).

The classification result (see table 6A in appendix) shows that one flower has been wrongly classified *virginica* when it is *a priori* a *versicolor* species.

The discriminant analysis classifies in base to the original variables (no reference is made to the *a priori* classification of species) and this classification shows that two errors have been made; one prediction error: one predicted *versicolor* originally classified *virginica* and another one predicted *virginica* which has been originally classified versicolor).

## 3. Traditional Clustering Methods

### *3.1. K-means Classification Method*

The *K*-means clustering method is a traditional method that reassigns cases by moving them to the cluster whose centroid is the closest to that case. Reassignment continues until every case is assigned to the cluster with the nearest centroid; such a procedure minimizes the variance within each cluster (Hartigan, J, 1975, Kaufman, L and Rousseuw, P, J, 1990).

This method is very close to Latent Structure Classification or to Classification with *Mixture* Modeling because both methods optimize some kind of criteria (Log Likelihood or Intra-class variance) for determining the proximity of cases to centroids and assign them to the closest group.

The *K*-means classification is less flexible in relation with the observed variables, in this case variables are mainly metric when the working variables used in Latent Method could be frequencies, ordinal, categorical or metrics or a combination of them. The latent solution is invariant to variables" transformation, so they do not need to be standardize.

The *K*-means method uses distance measures to compute determinist finite classes when the Latent Structure works with post hoc probabilities to compute more fuzzy classes based on Maximum Likelihood Methods (Lévy Mangin, Picon, 2006, 2004).

Table 7A (see appendix) summarizes the classification between *K*-means clustering method and the real flower *a priori* classification. This technique does not use any kind of training to classify data as the precedent techniques do; the results are much more erratic and there is some confusion in classifying 24 kinds of species predicted *versicolor* when they originally are *virginica* species.

The *K*-means clustering method (see table 8A in appendix) had some problems finding the real *virginica* classification; discriminant analysis only summarizes 26 *a priori* cases for this species if we compare with the real flower classification.

**Figure 3.** The multiplayer Perceptron with 1 hidden layer and 8 intermediate units.

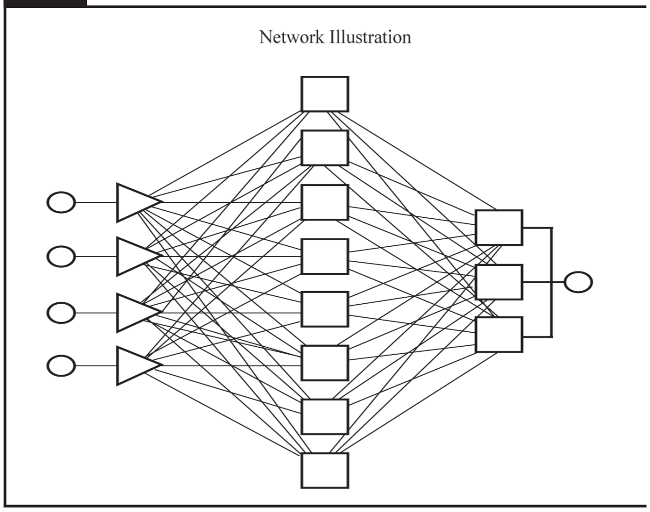
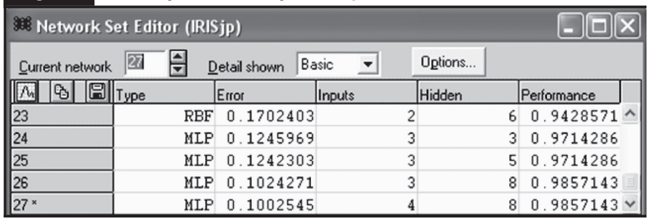



**Figure 4.** Accuracy of the Multilayer Perceptron Neural Network.


Network Set Editor (IRISjp)

Current network 27 Detail shown Basic Options...

| | Type | Error | Inputs | Hidden | Performance |
|---|---|---|---|---|---|
| 23 | RBF | 0.1702403 | 2 | 6 | 0.9428571 |
| 24 | MLP | 0.1245969 | 3 | 3 | 0.9714286 |
| 25 | MLP | 0.1242303 | 3 | 5 | 0.9714286 |
| 26 | MLP | 0.1024271 | 3 | 8 | 0.9857143 |
| 27 * | MLP | 0.1002545 | 4 | 8 | 0.9857143 |

### *3.2. Hierarchical Classification Method*

The Hierarchical method is also a traditional method based (in this case) on Euclidean distance between observations; we choose it as another reference method and, for comparison with the less traditional techniques.

The aggregation technique chosen is Average Linkage. Table 9A (see appendix) shows that *setosa* species have been perfectly classified (50, 100%), but it found 64 *versicolor* species instead 50 (128%), and 36 instead 50 *virginica* (which represents 72%).

The discriminant classification matrix (see table 10A in appendix) shows a poor classification in relation with the original flower species; which is satisfactory if we do not compare with a prior probability.

## 4. Discussion

Section 5 will discuss about the accuracy of all analyzed methods, we will compare methods using objective criteria of analysis as data training, Bayesian estimation and more classical methods, which, do not use these criteria. We will measure the classification (or posterior classification in comparison with the *a priori* or the original objective classification) results in relation with the *a priori* classification and we will make an overall estimation of the method based on specific criteria.

The *a priori* or original classification is not the perfect classification that everyone could suppose; based on a discriminant analysis classification three species have not been accurately classified. As we can see in table 11A (see appendix), two *versicolor* species have been classified as *virginica* species and one *virginica* species as *versicolor* species.

These three misclassified species could easily be observed in figure 5.

The final analysis of each method will be made first, in relation to the *a priori* (original) classification, and in a second stage based on the discriminant classification of cases (see table 12A in appendix). We will take in account that some methods have trained the data and others did not. The classification with Multilayer Perceptron trained 80 data over 150, the classification with *Mixture* Modeling 30 over 150, and all the other methods did not trained any data at all. The classification with Latent Class *Mixture* Modeling method did not trained any data but it is based on Bayesian Estimation, this method could be shown as one of the most accurate.

- The evaluation of each method based on a cross-tabulation between the classification method and the *a priori* classification shows that the most precise method is the method based on Neural Networks (Classification with Multilayer Perceptron); this method has been extensively trained in comparison with the classification with *Mixture* Modeling. The other methods have not been trained at all.

The classification with Multilayer Perceptron is not based on a Bayesian Estimation and does not compute an a posteriori probability of pertaining to one class in comparison with the two Bayesian methods of classification.

The classical methods are less interesting when an *a priori* classification is available, tables 8 and 10 suggest that *versicolor* and *virginica* species have been particularly misclassified.

- The classification of each method based on discriminant analysis suggests a better classification for the classical methods because there is not an *a priori* classification. The classification is made based on each method particularities. Nevertheless the methods based on Data Training and on Bayesian Estimation also suggest a very good classification of data (see tables 2A, 4A and 6A).

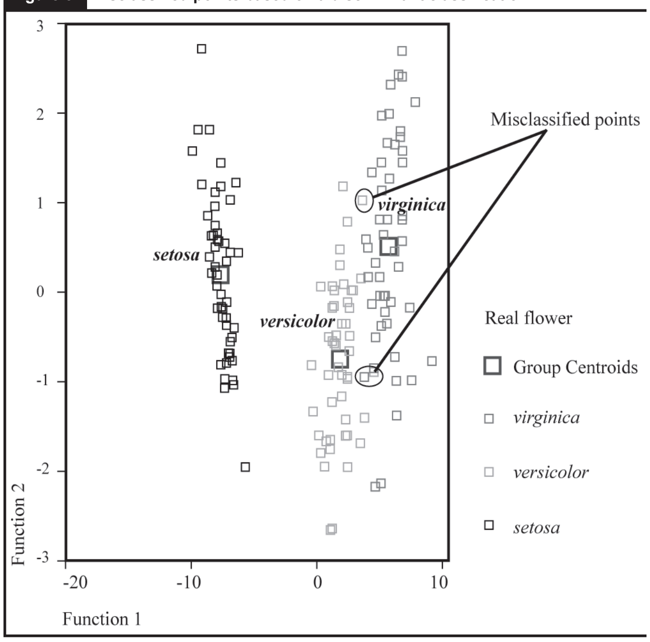
- As a global evaluation it would be very particularly accurate to choose the classification method based on the Neural Networks classification (Multilayer Perceptron), but there is a major inconvenient to generalize results, is that the training data sample is generally most important and extensive than for other methods (Bayesian methods for example). For that reason we cannot assign to this method (in our opinion) the first and an exclusive choice but we can consider it as one of the best. Our first choice will be the classification with *Mixture* Modeling and Training Data. In case of the availability of a previous *a priori* classification we do not suggest to consider the classical methods of classification (methods 4 and 5, table 12A).

## 5. Conclusion

Table 12 suggests that the best three methods for classifying data consider any form of data training or Bayesian estimation for the posterior classification. The traditional methods perform very poorly in relation with the *a priori* classification; the classical classification methods are simply less interesting (see table 12A in appendix) when a prior classification of data is made.

Models of *Mixture* modeling and Latent Structure analysis perform really well and are superior to Neural Networks when it comes to classify into groups with a minimum training of data; these methods are particularly interesting in the case of probabilistic or fuzzy classifications. These methods also permit to identify well-structured groups and an observation does not need to pertain exclusively to just one specific group. This corresponds to what Lilien and Rangaswamy said in 1998 to the effect that the consumer"s segments should be discrete (one consumer in one segment), overlapped (a consumer could pertain to two or more segments), or fuzzy (by assigning a probability to the consumer of pertaining to each group or cluster). To assign a consumer to an exclusive group is more attractive but overlapped and fuzzy segments are more realistic and theoretically and practically more precise. ergo

**Figure 5.** Misclassified points based on a discriminant classification.



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## Appendix

**Table 1A.** Cross tabulation of mixture modeling with correlation and training with real flower.

| | | | Real flower | | | Total |
|---|---|---|---|---|---|---|
| | | | *setosa* | *versicolor* | *virginica* | |
| Mixture modeling whit correlations and training | *setosa* | Count | 50 | 0 | 0 | 50 |
| | | % within mixture modeling with correlations and training | 100.0% | 0.0% | 0.0% | 100.0% |
| | | % within real flower | 100.0% | 0.0% | 0.0% | 33.3% |
| | | % of total | 33.3% | 0.0% | 0.0% | 33.3% |
| | *versicolor* | Count | 0 | 46 | 1 | 47 |
| | | % within mixture modeling with correlations and training | 0.0% | 97.9% | 2.1% | 100.0% |
| | | % within real flower | 0.0% | 92.0% | 2.0% | 31.3% |
| | | % of total | 0.0% | 30.7% | 0.7% | 31.3% |
| | *virginica* | Count | 0 | 4 | 49 | 53 |
| | | % within mixture modeling with correlations and training | 0.0% | 7.5% | 92.5% | 100.0% |
| | | % within real flower | 0.0% | 8.0% | 98.0% | 35.3% |
| | | % of total | 0.0% | 2.7% | 32.7% | 35.3% |
| Total | | Count | 50 | 50 | 50 | 150 |
| | | % within mixture modeling with correlations and training | 33.0% | 33.3% | 33.3% | 100.0% |
| | | % within real flower | 100.0% | 100.0% | 100.0% | 100.0% |
| | | % of total | 33.3% | 33.3% | 33.3% | 100.0% |

**Table 2A.** Mixture Modeling with correlation and training. Discriminant analysis classification matrix.

| | | Mixture modeling with correlations and training | Predicted group membership: *setosa* | *versicolor* | *virginica* | Total |
|---|---|---|---|---|---|---|
| Original | Count | *setosa* | 50 | 0 | 0 | 50 |
| | | *versicolor* | 0 | 47 | 0 | 47 |
| | | *virginica* | 0 | 2 | 51 | 53 |
| | % | *setosa* | 100.0 | 0.0 | 0.0 | 100.0 |
| | | *versicolor* | 0.0 | 100.0 | 0.0 | 100.0 |
| | | *virginica* | 0.0 | 3.8 | 96.2 | 100.0 |

Classification results[a]

a. 98.7% of original grouped cases correctly classified

**Table 3A.** Cross tabulation of Mixture Modeling without correlation and no training with Real Flower.

| | | | Real flower: *setosa* | *versicolor* | *virginica* | Total |
|---|---|---|---|---|---|---|
| Mixture modeling whitout correlations and training | *setosa* | Count | 50 | 0 | 0 | 50 |
| | | % within mixture modeling without correlations and training | 100.0% | 0.0% | 0.0% | 100.0% |
| | | % within real flower | 100.0% | 0.0% | 0.0% | 33.3% |
| | | % of total | 33.3% | 0.0% | 0.0% | 33.3% |
| | *versicolor* | Count | 0 | 47 | 4 | 51 |
| | | % within mixture modeling without correlations and training | 0.0% | 92.2% | 7.8% | 100.0% |
| | | % within real flower | 0.0% | 94.0% | 8.0% | 34.0% |
| | | % of total | 0.0% | 31.3% | 2.7% | 34.0% |
| | *virginica* | Count | 0 | 3 | 46 | 49 |
| | | % within mixture modeling without correlations and training | 0.0% | 6.1% | 93.9% | 100.0% |
| | | % within real flower | 0.0% | 6.0% | 92.0% | 32.7% |
| | | % of total | 0.0% | 2.0% | 30.7% | 32.7% |
| Total | | Count | 50 | 50 | 50 | 150 |
| | | % within mixture modeling without correlations and training | 33.0% | 33.3% | 33.3% | 100.0% |
| | | % within real flower | 100.0% | 100.0% | 100.0% | 100.0% |
| | | % of total | 33.3% | 33.3% | 33.3% | 100.0% |

**Table 4A.** Mixture Modeling without correlation and no training; Discriminant analysis classification matrix.

Classification results[a]

| | | Mixture modeling without correlations and training | Predicted group membership: *setosa* | *versicolor* | *virginica* | Total |
|---|---|---|---|---|---|---|
| Original | Count | *setosa* | 50 | 0 | 0 | 50 |
| | | *versicolor* | 0 | 51 | 0 | 51 |
| | | *virginica* | 0 | 1 | 48 | 49 |
| | % | *setosa* | 100.0 | 0.0 | 0.0 | 100.0 |
| | | *versicolor* | 0.0 | 100.0 | 0.0 | 100.0 |
| | | *virginica* | 0.0 | 2.0 | 98.0 | 100.0 |

a. 99.3% of original grouped cases correctly classified.

**Table 5A.** Cross tabulation of the Multilayer Perceptron Neural Networks classification with correlation and training with Real Flower. (Begins)

| | | | Real flower: *setosa* | *versicolor* | *virginica* | Total |
|---|---|---|---|---|---|---|
| Neural networks segmentation | *setosa* | Count | 50 | 0 | 0 | 50 |
| | | % within neural networks segmentation | 100.0% | 0.0% | 0.0% | 100.0% |
| | | % within real flower | 100.0% | 0.0% | 0.0% | 33.3% |
| | | % of total | 33.3% | 0.0% | 0.0% | 33.3% |

**Table 5A.** Cross tabulation of the Multilayer Perceptron Neural Networks classification with correlation and training with Real Flower. (Concludes)

| | | | Real flower | | | Total |
|---|---|---|---|---|---|---|
| | | | *setosa* | *versicolor* | *virginica* | |
| Neural networks segmentation | *versicolor* | Count | 0 | 49 | 0 | 49 |
| | | % within neural networks segmentation | 0.0% | 100.0% | 0.0% | 100.0% |
| | | % within real flower | 0.0% | 98.0% | 0.0% | 32.7% |
| | | % of total | 0.0% | 32.7% | 0.0% | 32.7% |
| | *virginica* | Count | 0 | 1 | 50 | 51 |
| | | % within neural networks segmentation | 0.0% | 2.0% | 98.0% | 100.0% |
| | | % within real flower | 0.0% | 2.0% | 100.0% | 34.0% |
| | | % of total | 0.0% | 0.7% | 33.3% | 34.0% |
| Total | | Count | 50 | 50 | 50 | 150 |
| | | % within neural networks segmentation | 33.0% | 33.3% | 33.3% | 100.0% |
| | | % within real flower | 100.0% | 100.0% | 100.0% | 100.0% |
| | | % of total | 33.3% | 33.3% | 33.3% | 100.0% |

**Table 6A.** Neural Networks discriminant analysis classification matrix.

| Classification Results[a] | | | | | | |
|---|---|---|---|---|---|---|
| | | | Predicted group membership | | | |
| Type | Statist ics | Neural networks segmentation | *setosa* | *versicolor* | *virginica* | Total |
| Original | Count | *setosa* | 50 | 0 | 0 | 50 |
| | | *versicolor* | 0 | 48 | 1 | 49 |
| | | *virginica* | 0 | 1 | 50 | 51 |
| | % | *setosa* | 100.0 | 0.0 | 0.0 | 100.0 |
| | | *versicolor* | 0.0 | 98.0 | 2.0 | 100.0 |
| | | *virginica* | 0.0 | 2.0 | 98.0 | 100.0 |

a. 96.7% of original grouped cases correctly classified.

**Table 7A.** Cross tabulation of ***K-Means*** classification with Real Flower.

| | | | Real flower | | | Total |
|---|---|---|---|---|---|---|
| | | | *setosa* | *versicolor* | *virginica* | |
| Qmeans clustering | *setosa* | Count | 50 | 0 | 0 | 50 |
| | | % within Qmeans clustering | 100.0% | 0.0% | 0.0% | 100.0% |
| | | % within real flower | 100.0% | 0.0% | 0.0% | 33.3% |
| | | % of total | 33.3% | 0.0% | 0.0% | 33.3% |
| | *versicolor* | Count | 0 | 50 | 24 | 74 |
| | | % within Qmeans clustering | 0.0% | 67.6% | 32.4% | 100.0% |
| | | % within real flower | 0.0% | 100.0% | 48.0% | 49.3% |
| | | % of total | 0.0% | 33.3% | 16 .0% | 49.3% |
| | *virginica* | Count | 0 | 1 | 26 | 26 |
| | | % within Qmeans clustering | 0.0% | 2.0% | 100.0% | 100.0% |
| | | % within real flower | 0.0% | 2.0% | 52.0% | 17.3% |
| | | % of total | 0.0% | 0.7% | 17.3% | 17.3% |
| Total | | Count | 50 | 50 | 50 | 150 |
| | | % within Qmeans clustering | 33.3% | 33.3% | 33.3% | 100.0% |
| | | % within real flower | 100.0% | 100.0% | 100.0% | 100.0% |
| | | % of total | 33.3% | 33.3% | 33.3% | 100.0% |

**Table 8A.** Discriminant Classification matrix for ***K-Means*** clustering method.

| Classification Results[a] | | | | | | |
|---|---|---|---|---|---|---|
| | | | Predicted group membership | | | |
| | | Qmeans clustering | *setosa* | *versicolor* | *virginica* | Total |
| Original | Count | *setosa* | 50 | 0 | 0 | 50 |
| | | *versicolor* | 0 | 67 | 7 | 74 |
| | | *virginica* | 0 | 0 | 26 | 26 |
| | % | *setosa* | 100.0 | 0.0 | 0.0 | 100.0 |
| | | *versicolor* | 0.0 | 90.5 | 9.5 | 100.0 |
| | | *virginica* | 0.0 | 0.0 | 100.0 | 100.0 |

[a] 95.3% of original grouped cases correctly classified.

**Table 9A.** Cross tabulation of the Multilayer Perceptron Neural Networks classification with correlation and training with Real Flower.

| Average linkage (Between Groups) * real flower crosstabulation | | | Real flower | | | Total |
|---|---|---|---|---|---|---|
| | | | *setosa* | *versicolor* | *virginica* | |
| Average linkage (Between Groups) | *setosa* | Count | 50 | 0 | 0 | 50 |
| | | % within Average linkage (Between Groups) | 100.0% | 0.0% | 0.0% | 100.0% |
| | | % within real flower | 100.0% | 0.0% | 0.0% | 33.3% |
| | | % of total | 33.3% | 0.0% | 0.0% | 33.3% |
| | *versicolor* | Count | 0 | 50 | 14 | 64 |
| | | % within Average linkage (Between Groups) | 0.0% | 78.1% | 21.9% | 100.0% |
| | | % within real flower | 0.0% | 100.0% | 28.0% | 42.7% |
| | | % of total | 0.0% | 33.3% | 9.3% | 42.7% |
| | *virginica* | Count | 0 | 0 | 36 | 36 |
| | | % within Average linkage (Between Groups) | 0.0% | 0.0% | 100.0% | 100.0% |
| | | % within real flower | 0.0% | 0.0% | 72.0% | 24.0% |
| | | % of total | 0.0% | 0.0% | 24.0% | 24.0% |
| Total | | Count | 50 | 50 | 50 | 150 |
| | | % within Average linkage (Between Groups) | 33.0% | 33.3% | 33.3% | 100.0% |
| | | % within real flower | 100.0% | 100.0% | 100.0% | 100.0% |
| | | % of total | 33.3% | 33.3% | 33.3% | 100.0% |

**Table 10A.** Discriminant Classification matrix for *K-Means* clustering method.

| Classification results[a] | | | Predicted group membership | | | Total |
|---|---|---|---|---|---|---|
| | | Average linkage (Between Groups) | *setosa* | *versicolor* | *virginica* | |
| Original | Count | *setosa* | 50 | 0 | 0 | 50 |
| | | *versicolor* | 0 | 59 | 5 | 64 |
| | | *virginica* | 0 | 0 | 36 | 36 |
| | % | *setosa* | 100.0 | 0.0 | 0.0 | 100.0 |
| | | *versicolor* | 0.0 | 92.2 | 7.8 | 100.0 |
| | | *virginica* | 0.0 | 0.0 | 100.0 | 100.0 |

[a]. 96.7% of original grouped cases correctly classified.

**Table 11A.** Discriminant Classification matrix for original classification method.

| Classification results[a] | | | Predicted group membership | | | Total |
|---|---|---|---|---|---|---|
| | | Real flower | *setosa* | *versicolor* | *virginica* | |
| Original | Count | *setosa* | 50 | 0 | 0 | 50 |
| | | *versicolor* | 0 | 48 | 5 | 64 |
| | | *virginica* | 0 | 1 | 36 | 36 |
| | % | *setosa* | 100.0 | 0.0 | 0.0 | 100.0 |
| | | *versicolor* | 0.0 | 96.0 | 4.0 | 100.0 |
| | | *virginica* | 0.0 | 2.0 | 98.0 | 100.0 |

[a] 98,0% of original grouped cases correctly classified.

**Table 12A.** Comparing performance of each classification method.

| Method of Classification | Data Training | Bayesian Estimation | Accuracy of Classification Results[1] (Crosstabulation) | Accuracy of the Discriminant Analysis Results[2] (Classification) | Overall evaluation of the method and choice |
|---|---|---|---|---|---|
| Mixture Modeling and training data | Yes | Yes | Very Satisfactory (*2nd classified*) | Very Satisfactory (*3rd classified*) | Excellent (*1rst classified*) |
| Latent Structure Analysis | No | Yes | Very Satisfactory (*3rd classified*) | Very Satisfactory (*2nd classified*) | Excellent (*2nd classified*) |
| Multilayer Perceptron | Yes | No | Very Satisfactory (*1rst classified*) | Very Satisfactory (*1rst classified*) | Excellent (*3rdclassified*) |
| K-means | No | No | Very Satisfactory (*5nd classified*) | Very Satisfactory (*5nd classified*) | Excellent (*5nd classified*) |
| Hierarchical method | No | No | Very Satisfactory (*4nd classified*) | Very Satisfactory (*4nd classified*) | Excellent (*4nd classified*) |

[1] Posterior classification in relation with a priori flower species classification (method based on cross-tabulation with original data).
[2] Classification in relation with the clustering method.